

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If L is a finite extension of k if k is finite extension of F , then prove that L is a finite extension of F . Moreover, $[L : F] = [L : K][K : F]$.
17. Show that let τ^* defines an isomorphism of $F[x]$ onto $F'[t]$ with the property that $\alpha\tau^* = \alpha'$ for every $\alpha \in F$.
18. Let k be the splitting field of $f(x)$ in $F[x]$ and let $P(x)$ be an irreducible factor of $f(x)$ in $F[x]$. If the roots of $P(x)$ are $\alpha_1, \dots, \alpha_n$ then prove that for each i there exists an automorphism σ_i in $G(k, F)$ such that $\sigma_i(\alpha_1) = \alpha_i$.
19. Show that a finite division ring is necessarily a commutative field.
20. Let D be a division ring algebraic over F , the field of real numbers, Then prove that D is isomorphic to one of : the field of real numbers, the field of complex numbers, or the division ring of real quaternions.

APRIL/MAY 2025

GMA21/DMA21 — ALGEBRA – II

Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define extension of field.
2. Write about minimal polynomial.
3. Determine the degrees of the splitting fields of the following polynomials over F . Then solve that $x^4 - 2$.
4. Discuss about splitting field over F .
5. Define Galois group.
6. If the following are polynomials in the elementary symmetric function x_1, x_2, x_3 , Then solve that $x_1^2 + x_2^2 + x_3^2$.
7. Prove that S_4 is a solvable group.



8. Write short note on Finite fields.
9. Brief about algebraic over a field F .
10. Define a joint of x .

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

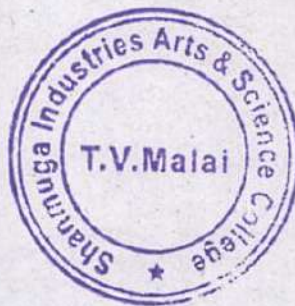
11. (a) If L is an algebraic extension of k and k is an algebraic extension of F , Then P.T. L is an algebraic extension of F .

Or

- (b) Prove that the mapping $\psi: F[x] \rightarrow F(a)$ defined by $h(x)\psi = h(a)$ is a homomorphism.
12. (a) If $p(x) \in F[x]$ and if k is an extension of F , then prove that for any element $b \in k$, $P(x) = (x - b)q(x) + P(b)$ where $q(x) \in k[x]$ and where $\deg q(x) = \deg P(x) - 1$.

Or

- (b) If F is the field of real numbers, prove that if ϕ is an auto morphism of F , Then ϕ leaves every element of F fixed.



13. (a) Show that the fixed of G is a subfield of k .

Or

- (b) Prove that if K is a normal Extension of F if and only if k is the splitting field of some Polynomial over F .

14. (a) Show that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

Or

- (b) If N is a normal subgroup of G prove that N' must also be a normal subgroup of G .

15. (a) Let C be the field of complex numbers and suppose that the divisions ring. D is algebraic over C . Then prove that $D = C$.

Or

- (b) Let H is a subring of Q . If $x \in H$ then show that $x^* \in H$ and $N(x)$ is a positive integer for every nonzero x in H .